RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER END SEM EXAMINATION, MARCH 2021

SECOND YEAR [BATCH 2019-22]

Date : 20/03/2021 **MATHEMATICS GENERAL**

Time : 11.00 am – 1.00 pm **Paper** : MAGT 3 Full Marks: 50

Instructions to the Candidates

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group: A

Geometry

Answer any **three** questions from question no. 1-5 in this group. [3 x 5 = 15 marks]

1. Reduce the equation $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$ to its canonical form and state the type of the conic. [5]
2. Show that $2x^2 + 3xy - 2y^2 + 7x - y + 3 = 0$ represents a pair of perpendicular straight lines. [5]
3. Find the locus of the points of intersection of pair of perpendicular tangents to the circle $x^2 + y^2 = 1$. [5]
4. Find the equation of $\frac{x^2}{16} - \frac{y^2}{9} = 1$ in the polar form with the left hand focus as the pole and the positive direction of the x-axis as the positive direction of the initial line. [5]

5. Find the locus of the vertices of the right circular cones that pass through the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ z=0$. [5]

Group: B

Algebra

Answer any **five** questions from question no. 6-12. [5 x 7 = 35 marks]

6. (a) Examine whether the composition 'o' defined on $\mathbb{Z}\times\mathbb{Z}$ defined by $(a,b)o(c,d)=(a-c,b-d)$; $(a,b),(c,d)\in\mathbb{Z}\times\mathbb{Z}$, is (i) commutative, (ii) associative. [2]

   (b) Let us consider the set H of all real 2×2 matrices $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : det\begin{pmatrix} a & b \\ c & d \end{pmatrix}=1\right\}$.
   Prove that (H,o) is a group where 'o' is the matrix multiplication. [4]

   (c) Find the number of generators of the group $(\mathbb{Z}_{15},+)$. [1]

7. (a) Let G be a cyclic group of order 30 generated by a. Find the order of the cyclic subgroup generated by a^{12}. [2]

   (b) Does $(n\mathbb{Z},+,.)$ form a ring, where '+' and '.' are two binary compositions defined on $\mathbb{Z}$? Justify your answer. [3]

   (c) Let $(\mathbb{R},+,.)$ be a ring. Define $b-c=b+(-c),\ b,c\in\mathbb{R}$.
   Prove that $a.(b-c)=a.b-a.c$. [2]

8. (a) Examine whether the set $\left\{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a,b,c\in\mathbb{R}\right\}$ forms a subring of $M_2(\mathbb{R})$ or not. [2]

   (b) Prove that $(\mathbb{Z}_7,+,.)$ is a finite field. [2]

   (c) Find the divisors of the zero and units of the ring $(\mathbb{Z}_9,+,.)$. [3]

9. (a) Examine if the set S is a subspace of $\mathbb{R}^3$, where $S=\{(x,y,z)\in\mathbb{R}^3 : x+y+z=0\}$. [2]

   (b) Find a basis and determine the dimension of the subspace S of the vector space $\mathbb{R}_{2\times2}$, where $S=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix}\in\mathbb{R}_{2\times2} : 2a+b=0\right\}$. [5]

10. (a) Extend the set S to obtain a basis of the vector space $\mathbb{R}^3$, where $S=\{(1,1,0),(1,1,1)\}$. [3]

    (b) Find the eigenvalues and the corresponding eigenvectors of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$. [4]

11. (a) Solve the equation $27x^3+42x^2-28x-8=0$, whose roots are in G.P. [3]

    (b) If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, find the values of (i) $\sum\frac{1}{\alpha^2}$, (ii) $\sum(\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma})$. [2+2]

12. (a) Find the maximum number of real roots of $x^5 + x^4 - 7x^3 - 22x^2 + x + 1 = 0$. [2]

(b) Solve: $x^6 - 1 = 0$. [3]

(c) Simplify: $\left(\dfrac{1 + \sin\frac{\pi}{8} + i\cos\frac{\pi}{8}}{1 + \sin\frac{\pi}{8} - i\cos\frac{\pi}{8}}\right)^8$. [2]

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